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### A DESIGN OF CONTROLLER WITH LOW GAIN INCLUDING CONSTRAINTS OF POLE PLACEMENT

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#### ABSTRACT

An iterative linear matrix inequalities (LMIs) algorithm is presented for centralized and decentralized state feedback controller designs. The controller is designed in such a way that places the closed loop poles under desired area and bounds near the boundary region with low gain controller. The application of algorithm is demonstrated through simulation studies of two-area power system model and formation control of unmanned aerial vehicles.

**Key words:** Pole, Design

#### INTRODUCTION

Decentralized control approach has been become the most popular and preferred control strategy for large scale system for over many past years [1,2]. The overview for analysis and solving methods for design problems can be found in [3]. In the decentralized control the whole system is considered as interconnection of subsystems. A local controller is designed for individual one based on available local information, which ensures stability and performance depending on one requirement| the local objective. The global objective is the designed local controllers must ensure the stability and the performance of the whole system.

The choice of subsystems affects the performance of control system, which is limited by information structure constraints [4,5]. Based on information structure constraints, decomposition of large scale system is the fundamental pre-requisite step for control designing for breaking a large dimensional system into smaller subsystems [6]. Control design for a system with overlapping subsystems is started by expanded the system into large dimensional, where the subsystems are appeared as disjoint [5, 6, 8]. The expanded space contains all the necessary information of the original system such that a control law is designed for each subsystem, then contracted back for implementation of control law into original system. For the expansion and contraction of the system, the mathematical framework is known as inclusion principle [4, 6].

#### Decentralized Controller

While dealing with control problems three steps: modeling, describing qualitative properties and controlling system behaviors are applied. This concept is applicable for centralized control, where a single controller is designed based on whole system information. But centralized control is not reliable and economical for the implementation into large scale system and also increases complexity in the design process. Because there is possibility of losing local data, presence of time delays due to long distance information transfer and presence of uncertainty in the model. Thus, the control problem becomes too large to be controlled and too complex to be solved.

#### Inclusion Principle

A large and complex system with overlapped subsystems can be expanded to a space in which subsystems appear as disjoint. In the expanded space a control law based on available information, is designed for each subsystem by using any standard method and then transformed it into a \_nal control law which is implementable into the original system. Consider two linear time invariant system

$$\mathbf{S}: \dot{x} = Ax + Bu, \quad x(t_0) = x_0$$

where  $x \in \mathbb{R}^n$  and  $u \in \mathbb{R}^m$  are the state and the control input of original system respectively.

$$\bar{\mathbf{S}}: \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}, \quad \bar{x}(t_0) = \bar{x}_0$$

1.6 Pole Placement in LMI regions

Stability is minimum requirement of any control system. But a good controller should also deliver a sufficient fast and well damped transient response which can be easily achieved by placing the closed loop poles under desired region D as shown in Fig. (1.2). Settling time and overshoot depend on the selection of damping ratio  $\cos\theta$  and speed of the system depends on  $\theta$ .

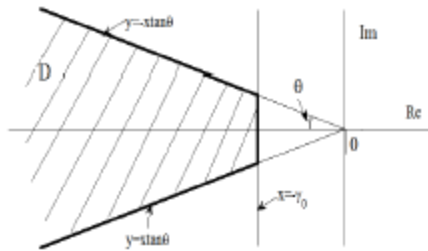


Figure 1.2: Desired Region in the Complex Plane

**Application Problems**

**Load Frequency Control**

In an interconnected power system power demand changes according to end users, this directly affects frequency and tie line power flow. The objectives of load frequency control (LFC) are to minimize the deviations in frequency and tie line power flow and to maintain steady state errors zero.

**Formation Control**

In formation control [25], a group of unmanned aerial vehicles move in a specified pattern, where may exist one or more leader and other followers. Different control strategies can be adopted depending on specific information structure constraints, to control the whole system.

**Design Algorithms**

This chapter presents design algorithms for both non-iterative and iterative case. And also presents design algorithm of homotopy method for solving decentralized control problem. In non-iterative algorithm the BMI problem is solved by converting it into convex optimization problem and taking additional constraints. For iterative algorithm the BMI problem is solved iteratively. In first step BMI problem is solved by converting it into convex optimization problem and taking additional constraints.

Solution of first step is taken as initial solution. In the second and third step BMI problem is solved by fixing one matrix variable and solving resulted LMI problem for other variable. In fourth step process is repeated until it results a low gain controller with required accuracy. Minimization of controller gain is achieved by taking the following constraints on P and K.

$$P < \beta I$$

$$K^T K = \alpha I$$

$$\begin{bmatrix} \alpha I & K^T \\ K & I \end{bmatrix} > 0$$

**Non-iterative Algorithm**

The following optimization problem is solved for  $P > 0$ , which gives controller gain K:

Subject to  $P > 0$

$$AP + PA^T + BY + Y^T B^T + 2\gamma_0 P < 0$$

$$\begin{bmatrix} \sin\theta [AP + PA^T + BY + Y^T B^T] & \cos\theta [AP - PA^T + BY - Y^T B^T] \\ \cos\theta [AP - PA^T + BY - Y^T B^T] & \sin\theta [AP + PA^T + BY + Y^T B^T] \end{bmatrix} < 0$$

Here K is simply obtained by  $K = YP^{-1}$ .

**Iterative Algorithm**

To develop iterative algorithm we use D-K type iteration algorithm. The following steps are followed one by one for calculation of optimal controller gain of the system:

1. Initialize iteration number  $i = 0$ . Solve for  $P > 0$  the following optimization problem.
2. Increase the iteration number by  $i = i + 1$  and assign  $K(i+1) = Y P(i+1)$ . Find a feasible solution for  $P > 0$  by solving the following optimization problem for given  $K(i+1)$  obtained from step 1

**Case Study 1: Load Frequency**

**Control of Power System**

Control of interconnected power systems is one of the most important issues on which many researches are going on. The main task of power system is to provide powers according to demand of connected various loads. As load changes, frequency and tie-line power flow are shifted from its nominal value. But, deviations in both should be zero. So the system requires load frequency control.

The primary task of LFC is to keep the frequency to its nominal value against the randomly varying loads, which also known as external disturbance. The secondary task is to regulate tie-line power flows between neighboring areas. On the other hand, increase in size and complexity of the power system introduces uncertainties and disturbances in control operation. Thus it is desired that the novel control strategies be developed to achieve LFC goals and

maintain reliability of the power system in an adequate level.

3.1 Model Description

The system model that we are going to use was derived in [20, 21]. Incremental changes in demand power arises two problems: \_rst control of real power and frequency, second control of reactive power and voltage. Both can be deal separately, here we will consider \_rst problem.

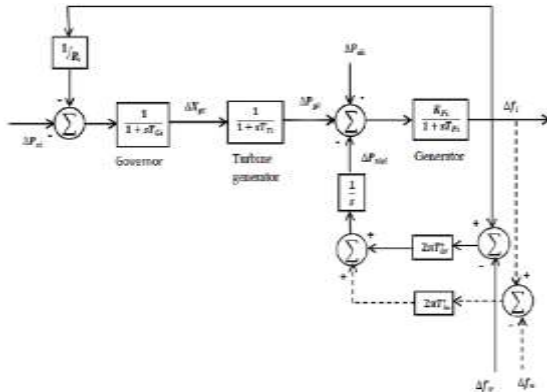


Figure 3.1: Block diagram of ith-area model

Two-Area Problem

For the two-area LFC problem the state vector, control input vector and perturbation are considered as

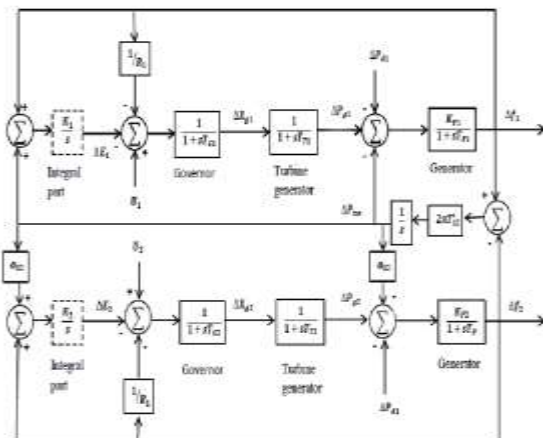


Figure 3.2: Two-Area Power System

Simulation Result:

In \_g.(3.3) the location of closed loop eigenvalues for both the case iterative and non-iterative are presented. We can observe that the closed loop poles in iterative algorithm are nearer than non-iterative. The control input u1 and u2 are presented in \_g.(3.4) for both algorithm. The simulation results for the responses of the system are shown in \_g.(3.5).

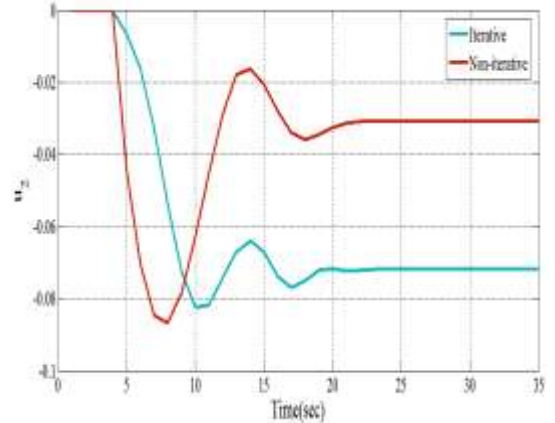
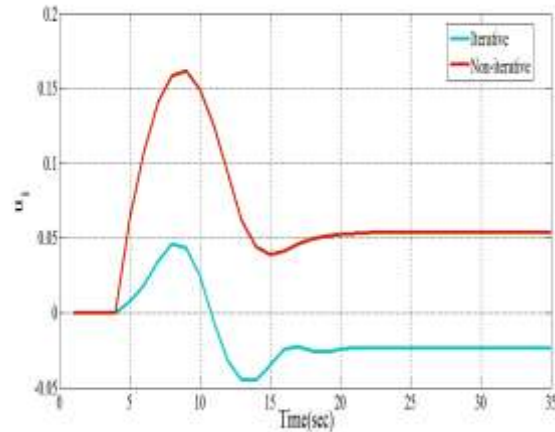
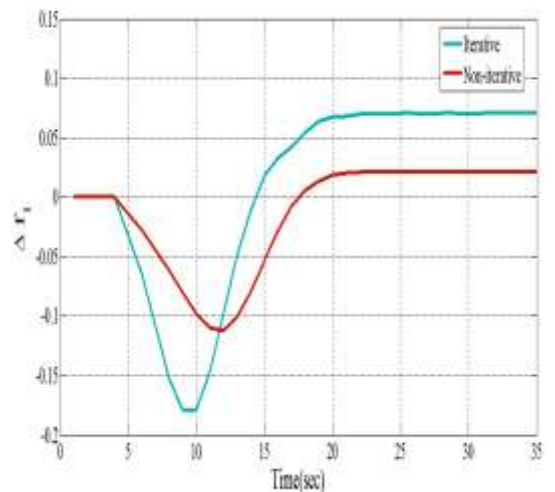


Figure 3.4: Control inputs u1 and u2(Centralized Control)



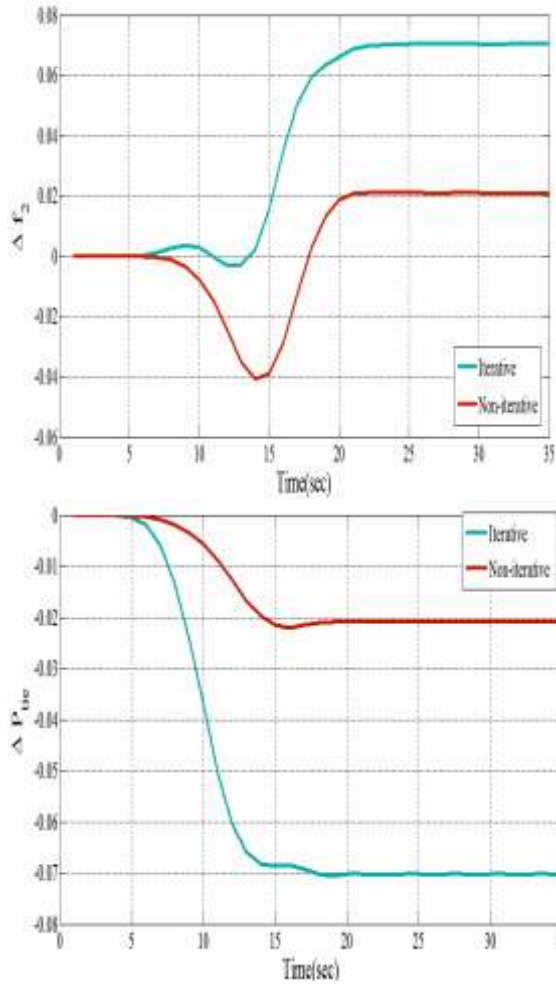


Figure 3.5: Responses of the system in the presence of a step disturbance input (Centralized Control)

The closed loop eigenvalues are placed at  $-28.69 - 58.24j$ ;  $-10.53 - 27.17j$ ;  $-12.9 - 9.24j$ ;  $-2.03 - 4.57j$  and  $-57.5$ . By observing the location of closed loop eigen values, we can conclude that the designed decentralized state feedback controller stabilizes the system.

For decentralized approach of controller design the closed loop eigen values are shown in Fig. (3.6) for both non-iterative and iterative algorithm. The control inputs  $u_1$  and  $u_2$  are given in Fig. (3.7) which are obtained by using controller gain  $KD_{nonit}$  and  $KD_{iter}$ . The responses of the system are shown in Fig. (3.8).

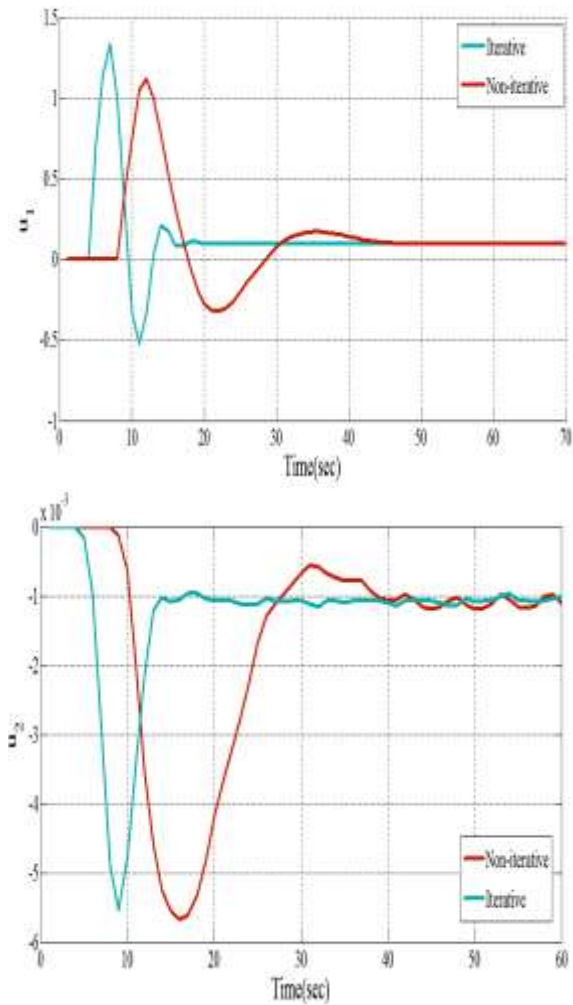
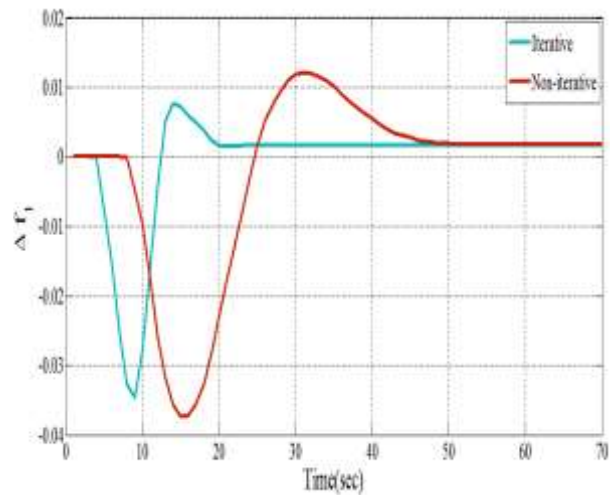


Figure 3.6: Control inputs  $u_1$  and  $u_2$  (Decentralized Control)



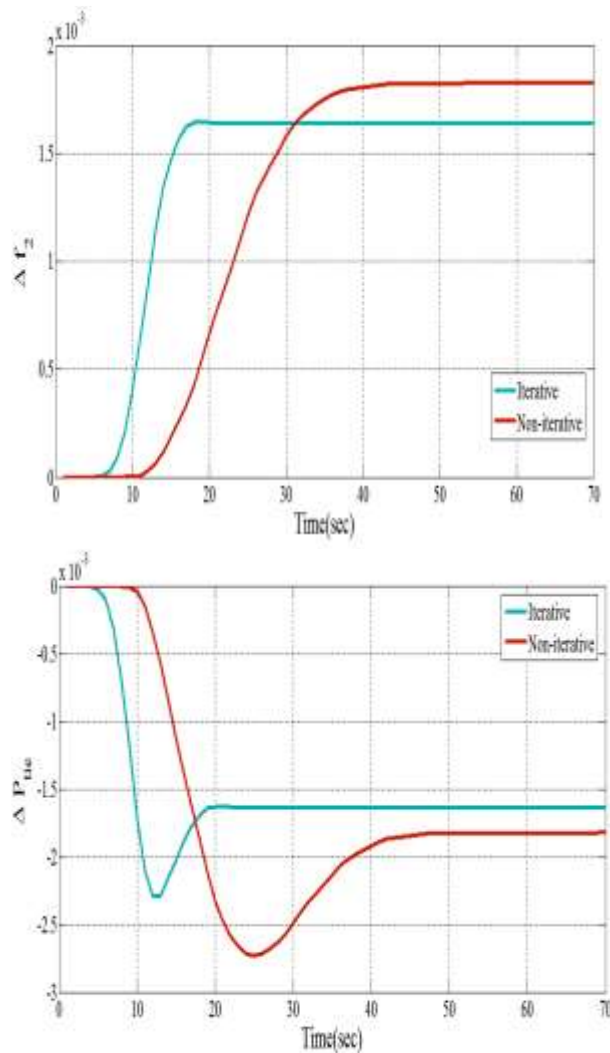


Figure 3.7: Responses of the system in the presence of a step disturbance input (Decentralized Control)

**4 Case Study 2: Formation control of unmanned aerial vehicles**

Formation control is one of the challenging problems in control engineering for controlling a group of unmanned aerial vehicles (UAVs). In many applications a group of UAVs follows a predefined trajectory while maintaining desired formation. There are three basic approaches for formation control: behavior-based, virtual structure and leader following. Here we are going to attempt leader following in which one or more vehicles may be leader while other followers.

Formation control has wide range of applications. For example, in military operations a group of AUVs are used for target vertical damage assessment and reconnaissance, in civilian works such as vegetation growth analysis. In automated highway system, the

efficiency of transportation network can be increased if the vehicles form a desired pattern at desired velocity while maintaining a specified distances between vehicles.

UAVs in formation is better than conventional systems, such as it can reduce system cost, increase the robustness and efficiency, and provide rapid configurability and structural flexibility (for decentralized control schemes).

This chapter describes decentralized approach of controller design.

The simulation result is shown in Fig. (4.2) for non-iterative algorithm by considering one set of initial conditions. Position co-ordinates are in feet.

Horizontal distances between vehicles V1 and V2, and V2 and V3 for non-iterative algorithm by considering one set of initial conditions are shown in Fig. (4.3). The distances are in feet. In Fig. (4.4), the errors in speed of vehicles V1, V2 and V3 are shown. Speeds are in [ft/s].

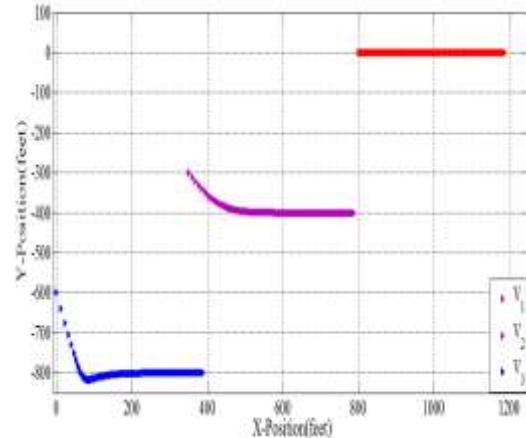
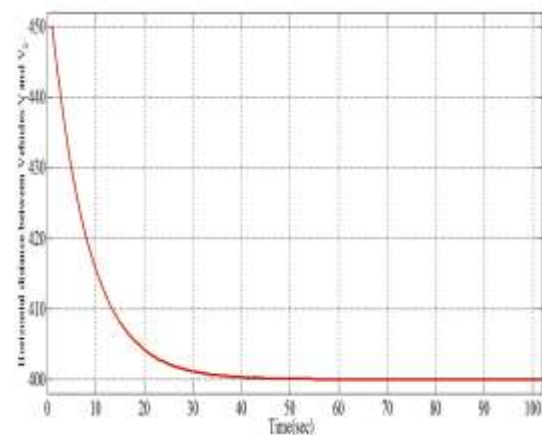


Figure 4.1: Snapshots of the formation for one set of initial conditions - Decentralized (Non-iterative algorithm)



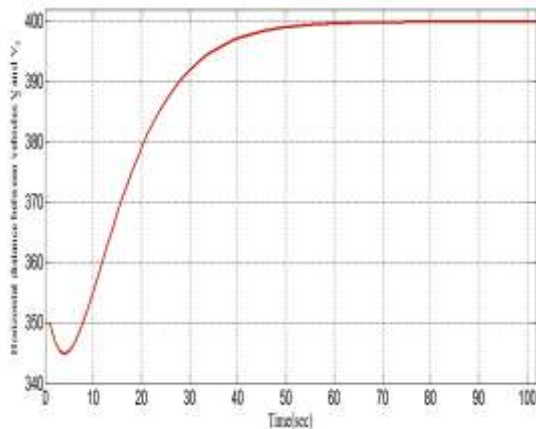


Figure 4.2: Horizontal distances between vehicles V1 and V2, and V2 and V3 (Noniterative)

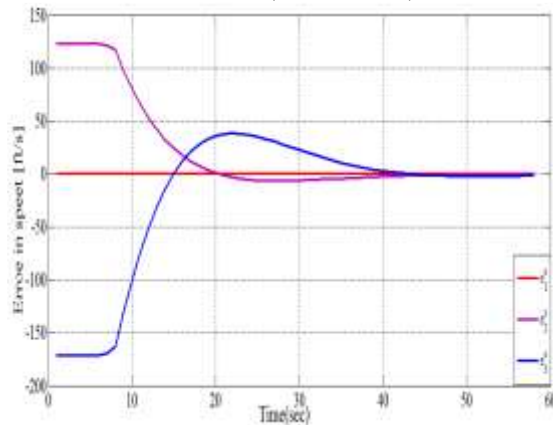


Figure 4.3: Error in speed of vehicles V1, V2 and V3 (Iterative algorithm)

## CONCLUSION

In this report, algorithms for designing centralized and decentralized control laws with the aid of pole placement have been discussed. The presented controller places closed loop eigen values near the boundary region with low gain. Basically, two cases have been discussed. In first case methods for designing centralized and decentralized controller with desired transient performance have been presented for interconnected power system based on pole placement and Lyapunov stability theory.

The formulated optimization problems are solved by using non-iterative and iterative algorithm. Then the comparative studies of both algorithms have been done by simulating two-area power system model. In second case a method for designing decentralized controller with desired transient performance for formation control of a group of unmanned aerial vehicles has been presented based on pole placement and Lyapunov stability theory. The formation is considered as an interconnected subsystem. The original system is expanded into a higher dimensional

space where subsystems are appeared as disjoint. In the expanded space static feedback law with pole placement constraints is designed for each subsystem and then contracted back for original system. The optimization problem is solved by using homotopy method for the whole system and then an iterative algorithm is employed to obtain a low gain controller.

We observe the following points:

- The closed loop eigen values are more near to the boundary region in iterative method.
- According to stability criteria, if the poles are shifted towards imaginary axis it reduces the stability of system. Thus the closed loop system becomes less stable in case iterative algorithm as compared to non-iterative. On the other hand, simultaneously iterative algorithm results reduction in controller gain which increases stability. Thus ILMI algorithm provides enough minimum stability, which a closed loop system requires.

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